

Higher-Dimensional QCD without the Strong CP Problem

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Abstract

QCD in a five-dimensional sliced bulk with chiral extra-quarks on the boundaries is generically free from the strong CP problem. Accidental axial symmetry is naturally present except for suppressed breaking interactions, which plays a role of the Peccei-Quinn symmetry to make the strong CP phase sufficiently small.

1 Introduction

The standard model of elementary particles has two apparent fine-tuning problems which are hard to be undertaken directly by additional (gauge) symmetries: the cosmological constant and the strong CP [1] problems. The presence of extra dimensions might serve as an alternative to symmetry in obtaining such fine-tuned parameters in a natural way.

In this paper, as a concrete example, we consider QCD in a five-dimensional sliced bulk with chiral extra-quarks on the boundaries and point out that it is generically free from the strong CP problem.

For definiteness, let us suppose that there is a pair of extra-quarks in addition to the standard-model quarks: a left-handed colored fermion ψ_L and a right-handed one ψ_R . We assume an extra-dimensional space which separates them from each other along the extra dimension. If the distance between them is sufficiently large, the theory possesses an axial U(1) symmetry

$$\psi_L \rightarrow e^{i\alpha}\psi_L, \quad \psi_R \rightarrow e^{-i\alpha}\psi_R \quad (1)$$

approximately, whose breaking is suppressed at a fundamental scale. This accidental global symmetry, which is actually broken by a QCD anomaly, plays a role of the Peccei-Quinn symmetry [2], making the effective strong CP phase to be sufficiently small.

The point is that the presence of such an approximate symmetry is not an artificial requirement, but a natural result stemming from the higher-dimensional space, which might well be stable even against possible quantum gravitational corrections.

2 Bulk color gauge theory

Let us consider a four-dimensional Minkowski spacetime M_4 along with one-dimensional extra-space S^1 , whose coordinate y extends from $-l$ to l (that is, two points at $y = l$ and $y = -l$ are identified). The $SU(3)_C$ gauge field is assumed to propagate on the whole spacetime $M_4 \times S^1$. The action of the five-dimensional gauge field is given by

$$S_A = \int d^4x \int_{-l}^l dy \frac{M_*}{4g^2(y)} \text{tr}(F_{MN}F^{MN}) + \int h(y) \text{tr} \left(AF^2 - \frac{1}{2}A^3F + \frac{1}{10}A^5 \right), \quad (2)$$

where $F_{MN} = \partial_M A_N - \partial_N A_M + [A_M, A_N]$ ($M, N = 0, \dots, 4; x^4 \equiv y$), $A = A_M dx^M$, and $F = dA + A^2 = (1/2)F_{MN}dx^M dx^N$. Here, M_* is supposed to be a cutoff scale in the

higher-dimensional gauge theory and $g(y)$ and $h(y)$ are gauge and Chern-Simons coupling functions, respectively.

Kaluza-Klein reduction to the four-dimensional spacetime, however, yields a massless color-octet scalar which is undesirable in the low-energy spectrum. Hence we consider an S^1/\mathbf{Z}_2 orbifold instead of the S^1 . The five-dimensional gauge field $A_M(x, y)$ is now under a constraint

$$A_\mu(x, y) = A_\mu(x, -y), \quad A_4(x, y) = -A_4(x, -y), \quad (3)$$

where $\mu = 0, \dots, 3$. Then, we have only a vector field at low energies without the scalar field after the Kaluza-Klein reduction. In order to define the theory on the orbifold consistently, the action eq.(2) on the S^1 should be invariant under the \mathbf{Z}_2 transformation. This invariance is achieved as long as $g(y) = g(-y)$ and $h(y) = -h(-y)$. We take the $g(y)$ to be y -independent and the $h(y)$ as

$$h(y) = c \frac{y}{|y|}, \quad (4)$$

where c is a constant to be determined in the next section. The gauge symmetries are left unbroken with the infinitesimal $SU(3)_C$ gauge transformation parameter restricted to satisfy $\varepsilon(x, y) = \varepsilon(x, -y)$.

3 Boundary extra-quarks

There are two fixed points in the S^1/\mathbf{Z}_2 orbifold: $y = 0$ and $y = l$. Let us put chiral extra-quarks¹ on the fixed-point boundaries: ² a left-handed extra-quark ψ_L at $y = 0$ and a right-handed extra-quark ψ_R at $y = l$. The action of the extra-quarks contains

$$S_\psi = \int_{y=0} d^4x \bar{\psi}_L i \not{D} \psi_L + \int_{y=l} d^4x \bar{\psi}_R i \not{D} \psi_R. \quad (5)$$

Under an infinitesimal gauge transformation, this fermionic sector provides a gauge anomaly

$$\delta S_{eff} = \frac{i}{24\pi^2} \int_{y=0} \text{tr} \left(\varepsilon d(AdA + \frac{1}{2}A^3) \right) - \frac{i}{24\pi^2} \int_{y=l} \text{tr} \left(\varepsilon d(AdA + \frac{1}{2}A^3) \right) \quad (6)$$

¹Instead of boundary extra-quarks, we may introduce vector-like extra-quarks in the bulk with a domain wall background, which results in a similar separation of chiral extra-quarks along the extra dimension.

²The standard-model quarks may be put on one of the fixed points, which does not alter the point in the following discussion.

due to its chirality, though the fermion content is vector-like from a four-dimensional perspective.

On the other hand, the bulk action yields

$$\begin{aligned}\delta S_A &= \int h(y) \operatorname{tr} \left((d\varepsilon) d(AdA + \frac{1}{2}A^3) \right) \\ &= - \int (dh(y)) \operatorname{tr} \left(\varepsilon d(AdA + \frac{1}{2}A^3) \right)\end{aligned}\tag{7}$$

under the gauge transformation. Gauge anomaly cancellation with the fermionic sector implies

$$c = \frac{i}{48\pi^2}.\tag{8}$$

4 Anomalous quasi-symmetry

The gauge-invariant theory is given by the total action $S = S_A + S_\psi$. Then, there is an approximate axial U(1) symmetry given by eq.(1). Indeed, interactions between the chiral extra-quarks (in the four-dimensional effective theory) such as

$$M_* \psi_L \psi_R^\dagger + \text{h.c.}\tag{9}$$

are suppressed by $e^{-M_* l}/(M_* l)$ with M_* as the cutoff scale at which new particles and interactions may arise (see also the discussion in the final section).

The extra-quarks should be decoupled from the low-energy spectrum to escape from detection. Thus, we introduce hypercolor gauge interactions in the bulk to confine the extra-quarks at high energies. Such new gauge interactions at the same time induce a chiral condensate $\langle \psi_L \psi_R^\dagger \rangle$, break down the axial U(1) symmetry and provide a corresponding Nambu-Goldstone (NG) boson called an axion [3]. Non-vanishing anomaly $U(1)[SU(3)_C]^2$ induces a potential of the axion field.

Let us adopt $SU(3)_H$ as the hypercolor gauge group and assume that the chiral fermions on each boundary transform as $\psi_L(\mathbf{3}, \mathbf{3}^*)$ and $\psi_R(\mathbf{3}, \mathbf{3}^*)$ under the $SU(3)_C \times SU(3)_H$ gauge group.³ The $SU(3)_H$ interactions are supposed to be strong and confining at an intermediate scale F_a , and a chiral condensate $\langle \psi_L \psi_R^\dagger \rangle \simeq F_a^3$ develops. Note that the $[SU(3)_H]^3$

³Extensions to larger gauge groups and fermion representations are straightforward, which are touched upon in the final section.

anomaly due to the fermionic sector can also be canceled by bulk Chern-Simons terms in a similar way as in the previous section.

$SU(3)_H$ -charged particles are confined and what is left at low energies are only massless NG bosons. When one switches off the $SU(3)_C$ gauge interactions (to concentrate on the strong dynamics of the $SU(3)_H$ interactions), then there is $U(3)_L \times U(3)_R$ flavor symmetry that acts on ψ_L and ψ_R^\dagger , where the $SU(3)$ subgroup of the diagonal $U(3)$ symmetry is actually gauged as $SU(3)_C$. The flavor symmetry $U(3)_L \times U(3)_R$ is spontaneously broken down to the diagonal $U(3)$ symmetry. The NG bosons due to this chiral symmetry breaking transform as $\mathbf{3} \times \mathbf{3}^* = \mathbf{adj.} + \mathbf{1}$ under the $SU(3)_C$. However, there is not $U(3)_L \times U(3)_R$ symmetry actually since the diagonal $SU(3)$ is gauged as the $SU(3)_C$ gauge group, and hence the adjoint-part of the NG bosons acquires masses due to the $SU(3)_C$ radiative corrections. What remains massless is only the color-singlet NG boson, which corresponds to the axial $U(1)$ symmetry in eq.(1).

The axial symmetry discussed above, however, also has $U(1)[SU(3)_H]^2$ anomaly. Therefore, the color-singlet NG boson obtains a large mass and it cannot play a role of the axion for the color $SU(3)_C$. Hence we further introduce an additional pair of chiral fermions on each boundary: $\chi_L(\mathbf{1}, \mathbf{3}^*)$ at $y = 0$ and $\chi_R(\mathbf{1}, \mathbf{3}^*)$ at $y = l$. The global symmetry is now $U(4)_L \times U(4)_R$ if the $SU(3)_C$ gauge interactions are neglected. The strong dynamics of the $SU(3)_H$ gauge interactions leads to spontaneous breakdown of the chiral symmetry, $\langle \psi_L \psi_R^\dagger \rangle \simeq F_a^3$ and $\langle \chi_L \chi_R^\dagger \rangle \simeq F_a^3$. Two color-singlets would remain massless if it were not for anomalies. In this case, one of them does play a role of the axion that makes the effective strong CP phase to be sufficiently small.

5 Discussion

The accidental chiral symmetry discussed above is broken by effective operators of the chiral fermions. The operators involving both ' ψ_L or χ_L ' and ' ψ_R^\dagger or χ_R^\dagger ', such as eq.(9), are highly suppressed.⁴ However, the operators involving either ' ψ_L and χ_L ' or ' ψ_R^\dagger and χ_R^\dagger ' are expected to be suppressed only by powers of $1/M_*$. Thus, such operators induce an additional potential of the axion, but this correction is not too large to render the

⁴ $M_* l \simeq 150 + 3 \ln(F_a/10^{12} \text{GeV})$ is sufficient.

Peccei-Quinn mechanism ineffective: Axial-symmetry breaking on each boundary may have coupling coefficients of order one. Such operators of the lowest mass dimension are given by

$$\int_{y=0} d^4x \frac{1}{M_*^5} (\psi_L)^3 (\psi_L)^3 + \int_{y=l} d^4x \frac{1}{M_*^5} (\psi_R^\dagger)^3 (\psi_R^\dagger)^3 + \text{h.c.} \quad (10)$$

Integration of heavy particles of masses of order F_a induces an additional potential of the axion field a as

$$V(a) \simeq \frac{F_a^{14}}{M_*^{10}} f\left(\frac{a}{F_a}\right), \quad (11)$$

where $f(a/F_a)$ is a function of a/F_a , whose minimum is generically different from that of the potential induced exclusively by the QCD effects. The resulting shift in the θ_{eff} parameter of the QCD is expected to be of order

$$\theta_{\text{eff}} \simeq \left(\frac{F_a}{M_*}\right)^{14} \left(\frac{M_*}{\Lambda_{\text{QCD}}}\right)^4 = 10^{-10} \left(\frac{F_a 10^5}{M_*}\right)^{14} \left(\frac{M_*}{\Lambda_{\text{QCD}} 10^{15}}\right)^4. \quad (12)$$

The present experimental bound $\theta_{\text{eff}} < 10^{-9}$ is satisfied, for instance, with $M_* \simeq 10^{18} \text{GeV}$ and $F_a \leq 10^{12} \text{GeV}$, which is phenomenologically viable.

We note that the axial-symmetry breaking operators on each boundary can be made to have higher mass dimensions if we adopt a larger hypercolor gauge group instead of the $\text{SU}(3)_H$. Then, the θ_{eff} will be more suppressed.

References

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